Rate maximization loading algorithm for LP-OFDM systems with imperfect CSI

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***Abstract*—The problem of bit rate maximization of a linear precoded orthogonal frequency division multiplexing (LP-OFDM) system is considered for imperfect channel state information. The discrete bit loading algorithms are proposed, which sustain the target bit error rate even under high mean square error (MSE) of estimation. The proposed scheme enhances the robustness of the system against noisy channel estimation without significantly compromising on the system throughput. The results are shown for a power line communication system using a well-known multipath channel model. It is shown that the proposed LP-OFDM allocation is more robust to estimation noise than OFDM allocations and provides higher throughputs even at high MSEs.**

***Index Terms*—Adaptive modulation, imperfect channel state information, linear precoded orthogonal frequency division multiplexing (LP-OFDM), power line communications (PLC), resource management.**

I. Introduction

In modern multicarrier systems, discrete bit loading [1] provides the capability to adaptively modulate different subcarriers, according to the signal-to-noise ratio (SNR) available on these subcarriers. The purpose of resource allocation is to optimize either the throughput or the robustness of the system. Under power spectral density (PSD) constraint and for a given error rate, the resource allocation generally gives either the maximum bit rate for a given system margin or the maximum system margin for a target bit rate. The former is a rate maximization (RM) problem [2] and the latter one is a margin maximization (MM) optimization problem [3], which is also known in the literature as the problem of power minimization under fixed bit and error rate.

Linear precoded orthogonal frequency-division multiplexing (LP-OFDM) is based on classical OFDM combined with a linear precoding component. The idea is to group together a set of subcarriers precoding sequences. Each resulting set accumulates the energies of all of its subcarriers to achieve an equivalent SNR such that the total number of bits supported is greater than the sum of the bits supported by each subcarrier individually. The aims of this scheme are to make the multicarrier system more flexible, with reduced limitations and improved overall system performance, without increasing the system complexity significantly. Although initially used for multi-user access schemes, the idea of combining linear precoding technique with multicarrier modulation can be extended to all single-user OFDM systems.

The resource allocation is generally performed on the transmitting side, where it is supposed that the channel has been perfectly estimated in advance and according to the channel responses on different subcarriers, the bit and power are allocated, such as various constellations of quadrature amplitude modulation (QAM) are used to assign different number of bits per subcarrier. In practice, perfect channel state information (CSI) is rarely achieved. The problem of imperfect CSI has already been discussed for OFDM systems in the literature [4], [5]. Studies on adaptive modulation based on imperfect CSI for multiple-input and multiple-output OFDM systems have also been performed [6]-[8]. In this paper, we consider the bit rate maximization problem for LP-OFDM systems, taking into account the imperfect CSI. The bit loading algorithms are proposed, which consider the estimation noise before allocating bit and power to different subcarriers. These algorithms underload the system for higher values of mean square error (MSE) of the estimator, in order to sustain an affordable value of mean BER. Whereas an algorithm which does not take into account the estimation errors, can overload the system and subsequently significantly increases the mean BER of the system.

The rest of the paper is organized as follows. In Section II, the structure of the LP-OFDM system is described. Section III explains the iterative allocation for OFDM systems without taking into account the estimation noise. The OFDM system is described in Section IV and an efficient bit loading algorithm is then proposed. In Section V, the issue of bit and power allocation for LP-OFDM systems is considered under imperfect CSI scenario. A bit and power allocation algorithm is also proposed here for LP-OFDM systems. In Section VI, simulation scenarios are discussed and results are presented for both the systems using a multipath PLC channel model. Finally, Section VII concludes the paper.

II. System Description

The combination of classical OFDM with a linear precoding component gives a general LP-OFDM system. This resulting LP-OFDM system is also known as spread-spectrum multicarrier multiple-access (SS-MC-MA) in mobile radio communications [9]. The classical system is modified by simply adding a precoding block in the transmission chain, which applies the precoding in the frequency dimension. Thus the system complexity is not significantly increased. Furthermore, the linear precoded component can be exploited to reduce the peak-to-average power ratio (PAPR) of OFDM

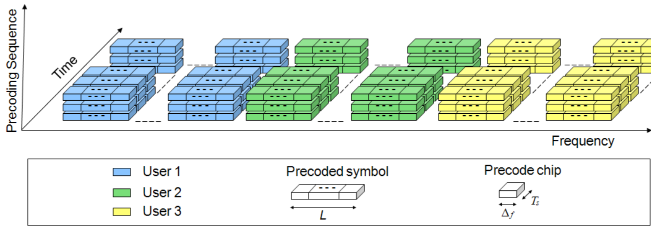

Fig. 1. LP-OFDM system description.

systems [10]. The linear precoding component improves the signal robustness against frequency selectivity and narrowband interference, since the signal bandwidth could become much larger than the coherence and interference bandwidths. It also accumulates the energies of many subcarriers by grouping them together which is useful in increasing the throughput especially under PSD constraint.

Fig. 1 shows the studied LP-OFDM system. The entire bandwidth is divided into N parallel subcarriers which are split up into K blocks S_k of L subcarriers, where k denotes the block number. The precoding function is then applied block-wise by mean of precoding sequences of length L, also known as precoding factor. Note that the subcarriers in a given block are not necessarily adjacent. Each user u of the network is being assigned a block B_u of subsets S_k. We emphasize that $\forall u,\ B_u$ are mutually exclusive subsets. Consequently, multiple access between the U users is managed following a frequency division multiple access (FDMA) approach, instead of a code division multiple access (CDMA) approach that is generally used in precoded systems, also called spreaded systems. In a general approach, the generated symbol vector at the output of the OFDM modulator for a single block LP-OFDM system can be written as

$$s = F^H M X. \tag{1}$$

Vector s is K-dimensional, with K the number of used subcarriers. $X = [x_1, \cdots, x_L]^T$ is the output of the serial-to-parallel conversion of the L QAM modulated symbols to be transmitted. M represents the precoding matrix of size $K \times L$ applied to X, which precodes L symbols over the K subcarriers. This precoding matrix is composed of orthogonal Hadamard matrices. Finally, F^H represents the Hermitian of the unitary Fourier matrix of size $K \times K$ that realizes the multicarrier modulation. It is worthy to mention here that we are going to consider the single user case only. This single user uses all the available precoding sequences of the system. The number of precoding sequences used to spread information symbols on one subset S_k is denoted by C^k, with $0 \leq C^k \leq L$, since we assume orthogonal sequences. A certain amount of energy E_c^k is assigned to each precoding sequence c^k associated to a given modulation symbol of b_c^k bits.

III. OFDM allocation without imperfect CSI consideration

For a known channel response, the number of bits b_i supported by subcarrier i can be given as [11]

$$b_i = \log_2\left(1 + \frac{E_s}{N_0}\frac{|H_i|^2}{\Gamma}\right) , \tag{2}$$

where $\frac{E_s}{N_0}$ is the SNR , H_i is the channel gain at subcarrier i, and Γ is the SNR gap. The SNR gap, Γ, for any uncoded QAM with a target symbol error rate, P, and for a null system margin, is given as [11]

$$\Gamma = \frac{1}{3}\left[Q^{-1}\left(\frac{P}{4}\right)\right]^2 , \tag{3}$$

where Q^{-1} is the inverse of the well-known Q-function. As it is clear from (3) that Γ depends upon the error rate of the system, and for a given error rate it has the same value for different modulation orders of QAM. In an allocation where effects of imperfect CSI are not considered the bits are allocated to different subcarriers in an iterative fashion by using (2), as Γ is known in advance. But in practice, where imperfect CSI is rarely achieved, the value of Γ which was calculated in advance remains no more valid. Therefore, the bits are wrongly allocated to the subcarriers, which results in the increase of BER at the receiver.

IV. OFDM allocation with imperfect CSI consideration

Practically speaking, it is impossible to obtain the perfect CSI. The classical allocations do not change the number of bits on a given subcarrier in case of noisy estimation. In the proposed algorithm, we underload the system, which means that less number of bits are allocated to various subcarriers in order to sustain the mean BER of the system. We propose an allocation, where without significantly compromising on the spectral efficiency, the robustness of the system is increased against the estimation noise. It is supposed that the estimation noise is included at the receiver and no noise is added when this information is sent back to the transmitter through a feedback channel. Therefore we have the same estimated channel at the both sides of the communication system. The well known power line channel is used for simulations and because of its quasi static nature delay in the CSI is neglected.

Since the channel estimation statistics depends on the estimation approach and the system details , therefore for the sake of simplicity, we characterize the estimation noise as additive Gaussian noise. The estimated channel gain is $\acute{H}_i = H_i + e_i$, where estimation noise is a complex Gaussian random variable with zero mean and a variance, σ_e^2, equal to the MSE of the channel estimator. It is also considered that $\acute{H}_i$ is the only known information about the current CSI of the i^{th} subcarrier[1]. The proposed approach uses the statistical information about the CSI errors to maintain the mean BER level and in result

[1]It should be noted here that channel quality indicator (CQI) is fed back to the transmitter instead of CSI, to reduce the feedback overhead.

reduces the spectral efficiency of the system. The expression of bit error rate $\acute{P}$, taking into account the imperfect channel estimation, can be given as [12]

$$\acute{P} = c_1 \frac{2^b - 1}{a + (2^b - 1)} \exp\left\{ -\frac{b}{a + (2^b - 1)} \right\} \ , \tag{4}$$

where

$$a = c_2 \frac{\sigma_e^2}{1+\sigma_e^2} \frac{E_s}{N_0} \ , \tag{5}$$

$$b = c_2 \left|s\right|^2 \frac{E_s}{N_0} \ , \tag{6}$$

and

$$s = \frac{1}{1+\sigma_e^2} \acute{H}, c_1 = 0.2, c_2 = 1.6 \ . \tag{7}$$

We consider an OFDM system with N subcarriers and the highest modulation order is limited to $2^{b_{\max}}$. The target bit error rate per subcarrier is expected to be 10^{-3}. The proposed algorithm for OFDM systems can be described as follows.

for i=1:N **do**
    Initiate $b_i = b_{\max}$
    Calculate $\acute{P}$ from (4)
    while $\acute{P} > 10^{-3}$ **do**
        $b_i = b_i - 1$
        Calculate $\acute{P}$ from (4)
    end while
end for

In this way, we obtain an allocation, which takes into account the effects of imperfect CSI. The system is underloaded for higher values of MSE to sustain a mean BER of the system.

V. LP-OFDM allocation with imperfect CSI consideration

The rate maximization problem for LP-OFDM systems has been discussed in the existing literature (for example [2], and [13]) without taking into account the effects of imperfect CSI. Here, we propose a bit and power allocation algorithm for LP-OFDM systems that maximizes the bit rate of the system for a given error rate and a defined PSD limit but also takes into account the noisy channel estimation. The expression of bit error rate $\acute{P}_c^k$ for a precoding sequence c in a given block k of length L, taking into account the imperfect channel estimation, can be given as

$$\acute{P}_c^k = c_1 \frac{2^{\frac{R_k}{L}} - 1}{a + \left(2^{\frac{R_k}{L}} - 1\right)} \exp\left\{ -\frac{b}{a + \left(2^{\frac{R_k}{L}} - 1\right)} \right\} \ , \tag{8}$$

where

$$a = c_2 \frac{\sigma_e^2}{1+\sigma_e^2} \frac{E_k}{LN_0} \ , \tag{9}$$

$$b = c_2 \frac{E_k}{N_0} \sum_{i \in S_k} \frac{1}{\left|s_i\right|^2} \ , \tag{10}$$

and

$$s = \frac{1}{1+\sigma_e^2} \acute{H}, c_1 = 0.2, c_2 = 1.6 \ , \tag{11}$$

where R_k and E_k are the total number of bits and the total transmit power available to block k, respectively. Here, we use the optimal allocation obtained in [13] for infinite granularity[2] of modulation. Thus, $b_c^k = \frac{R_k}{L}$ and $E_c^k = \frac{E_k}{L}$, where b_c^k and E_c^k are the number of bits and transmit power allocated to precoding sequence c, respectively. We propose a bit and power loading algorithm for practical systems (i.e. finite granularity of modulation). In this approach, firstly the number of bits per block are found in an iterative fashion and then these bits are distributed, among different precoding sequences of a given block, in the following manner:

$$b_c^k = \begin{cases} \lfloor R_k/L \rfloor + 1 & (1 \le i \le n_c^k \\ \lfloor R_k/L \rfloor & (n_c^k < i \le L) \end{cases} \tag{12}$$

where

$$n_c^k = \lfloor L(2^{R_k/L - \lfloor R_k/L \rfloor} - 1) \rfloor . \tag{13}$$

After deciding the suitable number of bits for all the precoding sequences, the transmit power is divided among the precoding sequences in the following manner:

$$E_c^k = \frac{(2^{b_c^k} - 1)}{\sum_{c=1}^{L} (2^{b_c^k} - 1)} E^k . \tag{14}$$

We consider an LP-OFDM system with N subcarriers, precoding factor L and number of blocks K. The highest modulation order is limited to $2^{b_{\max}}$ and the target bit error rate per block is expected to be 10^{-3}. The proposed algorithm for LP-OFDM systems can be described as follows

for i=1:K **do**
    Initiate $R_k = Lb_{\max}$
    Calculate $\acute{P}_c^k$ from (8)
    while $\acute{P}_c^k > 10^{-3}$ **do**
        $R_k = R_k - 1$
        Calculate $\acute{P}_c^k >$ from (8)
    end while
    Calculate n_c^k from (13)
    Calculate b_c^k from (12)
    Calculate E_c^k from (14)
end for

This algorithm allocates bit and power to various precoding sequences of the LP-OFDM system, taking into account the effects of noisy channel estimation. It underloads the system for higher MSEs to sustain a mean BER of the system and fairly increases the system robustness against imperfect CSI without significantly compromising on the system throughput.

[2]Infinite granularity corresponds to modulation orders, which are not necessarily integers.

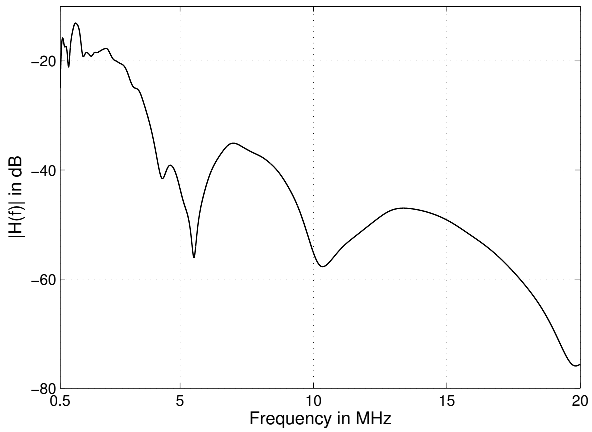

Fig. 2. 15-paths reference channel model for PLC [14]

TABLE I
PARAMETERS OF THE 15-PATH MODEL.

attenuation parameters					
$k=1$		$a_0=0$		$a_1=2.5\times10^{-9}$	
path-parameters					
p	g_p	d_p(m)	p	g_p	d_p(m)
1	0.029	90	9	0.071	411
2	0.043	102	10	−0.035	490
3	0.103	113	11	0.065	567
4	−0.058	143	12	−0.055	740
5	−0.045	148	13	0.042	960
6	−0.040	200	14	−0.059	1130
7	0.038	260	15	0.049	1250
8	−0.038	322			

VI. SIMULATIONS AND RESULTS

In this section, we will present simulation results for both the proposed allocation schemes. The results will be compared with iterative OFDM allocation, which does not take into account the noisy channel estimation. We use the multipath model for the power line channel, proposed in [14] and presented in Fig. 2. The considered reference model is 110 m link 15-paths model whose frequency response is given by

$$H(f)=\sum_{p=1}^{15} g_p\cdot e^{-(a_0+a_1 f^k)^{d_p}}\cdot e^{-j2\pi f\tau_p}, \qquad (15)$$

where τ_p is the delay of path p. The parameters of the 15-path model are listed in Table I. The generated signal is a baseband signal produced by dividing 20 MHz band of Zimmermann channel into 1024 subcarriers. It is assumed that the synchronization has been successfully performed. A background noise level of -93 dBm/Hz is assumed and the signal is transmitted with respect to a flat PSD of -40 dBm/Hz. The precoding factor L for LP-OFDM is 32 while the highest modulation order is limited to 2^{15}, as in very high speed digital subscriber line systems [15]. The target bit error rate per block is expected to be 10^{-3}. A complete communication system chain is simulated and the mean BER is calculated assuming the characteristics of channel estimator to be known.

Fig. 3 shows the Mean BER performance of all three allocations for different values of MSE. It can be observed that the proposed allocations are robust against the estimation noise and provide sustainable BER performance as compared to iterative OFDM allocation, which does not take into account the effects of imperfect CSI. It may further be seen that the proposed LP-OFDM allocation outperforms both the OFDM allocations and provides reasonable BER performance even at much higher values of MSE. For instance, iterative OFDM allocation without imperfect CSI consideration, for MSEs higher than 8×10^{-6} results in mean BERs higher than 10^{-3}, while for an MSE $=4\times10^{-5}$ the mean BER is around 10^{-1}.

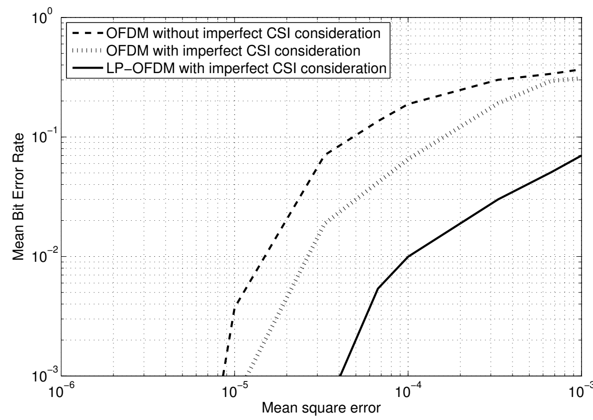

Fig. 3. Mean BER comparison for different values of mean square error.

The proposed LP-OFDM allocation provides a mean BER of 10^{-2} even at an MSE of 1×10^{-4}.

Fig. 4 compares the throughput performance of all three allocations for different values of MSE. The proposed allocations perform better than the classical iterative OFDM allocation for lower MSEs but underloads the system for higher values of estimation noise variance, as was expected. The classical iterative OFDM is achieving higher bit rate for an MSE greater than 8×10^{-6}, but as discussed earlier the mean BER perfomance of this allocation is collapsing for these MSEs. Thus these higher throughputs are of no use. Instead it is more useful to underload the system at this stage to maintain the mean BER performance and it is exactly what our proposed allocations are doing. At lower MSEs, The proposed LP-OFDM allocation provides much higher bit rate than OFDM allocations as it accumulates the energies of multiple subcarriers by grouping them together. Therefore, the proposed LP-OFDM allocation is utilizing the available energy more efficiently than OFDM allocations. Even under higher values of MSE, it is providing significantly sustainable

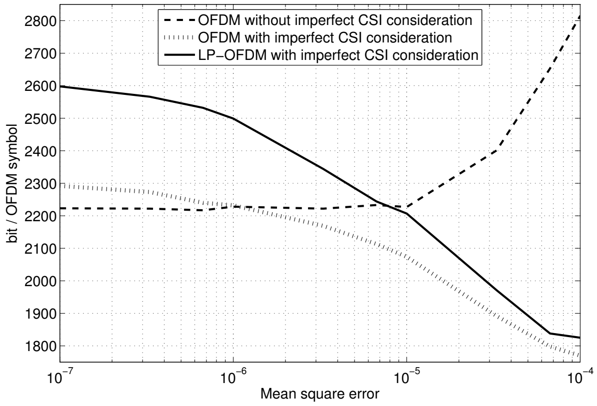

Fig. 4. Bit rate comparison for different values of mean square error.

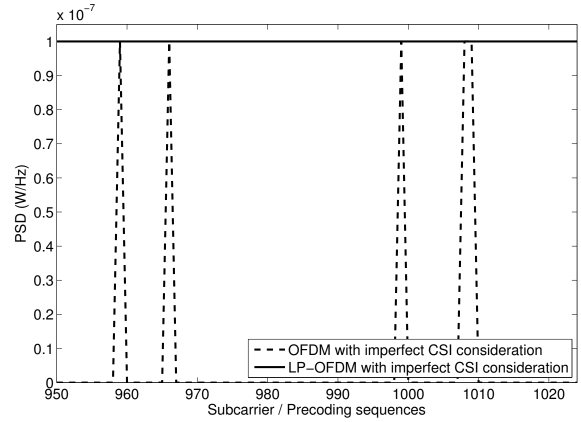

Fig. 5. Energy distribution comparison.

BER performance with fairly reasonable bit rate. The reason of this better performance can be easily understood from energy distribution diagram.

Fig. 5 compares the energy distribution of OFDM and LP-OFDM allocations for higher frequency range. The spike-shaped curve of the OFDM allocation shows the transitions of the modulation orders when no more energy is available to sustain the target BER. Under PSD constraint, the major task is to efficiently utilize the available energy, since the transmit power that is not used by a subcarrier (or a group of subcarriers) is lost and can not be used by other subcarriers (or groups of subcarriers). The LP-OFDM allocation accumulates the energies of all the subcarriers of the group to utilize it more efficiently, which can be seen in the greater area under the curve. It can be observed that it is using more energy than OFDM, utilizing almost all the energy available for the system. It accumulates the energies of different subcarriers to transmit more bits. Thus higher bit rates are obtained.

VII. Conclusion

In this paper, we considered the rate maximization problem for LP-OFDM systems taking into account the effects of imperfect CSI. We proposed two loading algorithms. One for classical OFDM systems and the other for LP-OFDM systems. It was observed that the proposed allocations provide significant robustness against noisy channel estimation. The proposed LP-OFDM system provides even better mean BER performance without significantly compromising on bit rate. It was also shown that LP-OFDM allocation utilizes the available energy more efficiently and thus provides much higher bit rates for lower values of MSE. We conclude that the proposed allocation provides sustainable mean BER performance with reasonable bit rate at higher estimation noise variance and improved bit rate at lower MSEs.

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